

# An analytic technique for the estimation of the light yield of a scintillation detector.

E. Segreto<sup>\*</sup>

*INFN - Laboratori Nazionali del Gran Sasso, Assergi, Italy*

## Abstract

A simple model for the estimation of the light yield of a scintillation detector is developed under general assumptions and relying exclusively on the knowledge of its optical properties. The model allows to easily incorporate effects related to Rayleigh scattering and absorption of the photons. The predictions of the model are benchmarked with the outcomes of Monte Carlo simulations of specific scintillation detectors. An accuracy at the level of few percent is achieved. The case of a real liquid argon based detector is explicitly treated and the predicted light yield is compared with the measured value.

## 1 Introduction.

A typical scintillation detector is constituted by a scintillating material contained in a reflective box and by a system of one or more Photo-Sensitive Devices (PSDs) that *observes* the active medium. This kind of detectors is widely used in many fields of physics to detect particles or energetic photons ([1], [2], [3]). In many cases they also allow to perform calorimetric measurements since their output signal is often proportional to the energy that the ionizing radiation leaves inside the active medium. The constant ratio between the signal (usually in charge) and the deposited energy is the light yield (LY) and is measured in (photo-) electrons/keV. The main factors that determine the LY of a detector are the abundance of photons produced per unit of deposited energy (photon yield), the optical properties of the scintillator and of the internal surface of the box where it is contained, the number and dimensions of the PSDs and their efficiency in converting photons into a detectable signal. The LY is one of the parameters that more deeply influences the design of a scintillation detector since, for each value of deposited energy, it fixes the scale of the intensity of the output signal. Therefore the layout of the detector needs to be optimized in order to match as well as possible the LY with the characteristics (energy, Linear Energy Transfer, ...) of the incoming radiation and of the electronic read-out chain. In the field of low energy particle physics (neutrino

---

<sup>\*</sup>E-mail: ettore.segreto@lngs.infn.it

physics, direct Dark Matter search, double  $\beta$  decay experiments, ...), for instance, where high sensitivity and high energy resolution is required the optimal LY value is often the highest achievable, compatibly with the others experimental constraints. Traditionally the LY is evaluated by means of Monte Carlo simulations. This technique has the advantage of giving very precise results at the expense of programming and mainly running codes that typically invoke the propagation of millions of photons and this can result extremely cumbersome especially in an optimization process. In this work an alternative and completely analytic approach for the estimation of the LY of simple detectors is presented. It offers the possibility to obtain fast and robust results, with an accuracy of few per cent if compared with the classical Monte Carlo approach. Furthermore the explicit dependence of the LY from all the optical parameters makes this technique particularly suitable for the process of detector optimization.

## 2 Light Yield calculation.

This work addresses the most simple and common layout for a scintillation detector: an uniform scintillating medium (in liquid, gaseous or solid state) is contained in a cell with (highly) reflecting internal surfaces and is observed by one ore more PSDs whose window(s) are installed on the internal surface of the cell. The following hypotheses are assumed to hold:

- The scintillator is uniform and completely fills the cell where it is contained. It is monochromatic. This is not a limitation for the model because in the case of a non monochromatic scintillator one can calculate the LY as a function of the wavelength,  $\lambda$ , of the emitted radiation and average it over the entire spectrum, that is:

$$LY = \frac{\int_0^\infty LY(\lambda) I(\lambda) d\lambda}{\int_0^\infty I(\lambda) d\lambda} \quad (1)$$

The scintillator is initially considered to be perfectly transparent to scintillation radiation, that is photons do not suffer absorption or elastic scattering (Rayleigh scattering) processes along their propagation. This assumption will be removed in section 2.2.

- The LY does not depend on the point where the ionizing radiation leaves its energy. This implies that the scintillating medium is contained in a cell of regular shape (sphere, square cylinder, cube...) and that its internal surface is reflective and its reflectivity  $R$  is high.
- Each PSD is schematized as: a window with given optical properties (reflectivity and transmissivity) coupled to a "*black box*" that absorbs all transmitted photons and produces a signal with a certain efficiency.<sup>1</sup>

---

<sup>1</sup>In the case of a photomultiplier this is not directly the Quantum Efficiency, because the latter includes in its definition effects related to window reflectivity and transmissivity and photo-cathode

The reflectivity of the internal surface of the cell and of the PSD window (and consequently its transmissivity) typically depends on the angle of impact,  $\theta$ , of the photon with respect to the normal to the surface. In this work only average values are considered. Assuming an uniform lambertian illumination the average reflectivity of a flat infinitesimal element of surface can be calculated as:

$$R = \int_0^{\pi/2} R(\theta) 2 \sin\theta \cos\theta d\theta \quad (2)$$

the same holds for the transmissivity.

## 2.1 Basic calculation.

The LY of this kind of scintillation detector can be factorized into three terms:

$$LY = N_\gamma \times \epsilon_{opt} \times \epsilon_{PSD} \quad (3)$$

where:

- $N_\gamma$  is the *photon yield* of the scintillator: it is the number of photons produced per unit of deposited energy by a certain radiation (usually in photons/keV);
- $\epsilon_{opt}$  is the *optical efficiency*: it is the fraction of the originally produced photons that manages to cross the windows of the PSDs. It depends on the optical properties of the boundary surface of the detector, of the scintillation medium and of the PSDs' windows;
- $\epsilon_{PSD}$  is the *conversion efficiency* of the PSDs: it is the efficiency of the PSD system in converting photons into signal (*photo-electrons*).

The dimensions of the  $LY$  are *photo-electrons/keV*.  $N_\gamma$  and  $\epsilon_{PSD}$  are characteristic parameters of the scintillator medium and of the photo-sensitive devices and in the majority of the cases are precisely known. On the other side  $\epsilon_{opt}$  is typically unknown and needs to be estimated. It represents the average probability that a scintillation photon produced in the active medium by an energy release reaches and crosses the window of one of the PSDs, surviving to the processes that can kill it while bunching inside the detector.

The propagation of photons inside a scintillation detector is an intrinsically recursive process. Consider, for example, a sphere containing a scintillating medium and assume that a fraction  $f$  of its internal surface is occupied by the window of a PSD, that is perfectly transparent to scintillation radiation and with refractive index matched to that of the scintillator. Neglect, for now, absorption and elastic scattering phenomena.

---

reflectivity. In the present schematization these effects are all transferred to the window and the PSD (photomultiplier) efficiency is the probability that a photon absorbed by the photo-cathode (black box) produces a photo-electron.

A photon produced in a random point inside the sphere and with a random direction when reaches the boundary surface has an average probability  $f$  to be detected (assuming  $\epsilon_{PSD} = 1$ ), since its impact point is uniformly distributed on the sphere. On the other hand it has a probability  $(1 - f)$  to hit the non active surface and if its reflectivity  $R$  is not zero it is sent back inside the scintillator with probability  $R(1 - f)$ . Reflected photon has again a random direction and a random production point (on the surface of the sphere this time) and has again a detection probability equal to  $f$  and a probability to be reflected equal to  $R(1 - f)$ . The same situation will repeat again identical to itself after any reflection.

Let's generalize these ideas and consider a general scintillation detector. In order to estimate its optical efficiency  $\epsilon_{opt}$  assume that the process that starts with the production and ends with the absorption/detection of the photon can be treated in a recursive way. This means that it can be divided into a series of subsequent and indistinguishable steps and that it is possible to define two quantities,  $\alpha$  and  $\beta$ , where:

- $\alpha$  is the *average* probability *per step* that a photon randomly generated in the scintillator volume (for the first step) or surviving from the previous step is **detected**.<sup>2</sup>
- $\beta$  is the *average* probability *per step* that a photon is **regenerated**, that is the probability that it is not *lost* (detected or absorbed) and that some physical process randomizes again its direction (reflection for instance).
- $\alpha$  and  $\beta$  are constant for all the steps (from the assumption on the recursiveness of the process).
- $\alpha \leq 1$  and  $\beta < 1$ .

With these assumptions it is easy to calculate the detection and regeneration probabilities for a photon at step  $n$  *after surviving to the previous  $n - 1$  steps*. The values are shown in table 1. Hence the optical efficiency, that is the sum of these detection probabilities over all the steps can be simply calculated as the sum of a geometric series:

$$\epsilon_{opt} = F(1, \alpha, \beta) = \sum_{n=0}^{\infty} \alpha \beta^n = \frac{\alpha}{1 - \beta} \quad (4)$$

This series converges because  $\beta < 1$ . The notation  $F(Q, \alpha, \beta)$  will be fully clear in the next section. In principle, for each given detector, one could define the elementary step in many different ways and this can make the calculations more or less difficult, but the final result is general and absolutely independent of the step definition.

Consider again the simple spherical scintillation detector described above. The step can be defined in a natural way as the photon propagation between subsequent interactions with the boundary surface: it starts just after one reflection and ends when photon

---

<sup>2</sup>Hereafter for *detected* photons we mean photons that succeed in crossing the PSD window.

Table 1: *Detection and regeneration probabilities for a photon propagating inside a scintillation detector as a function of the propagation step (see text).*

	detection probability	regeneration probability
step 0	$\alpha$	$\beta$
step 1	$\alpha\beta$	$\beta^2$
step 2	$\alpha\beta^2$	$\beta^3$
$\dots$	$\dots$	$\dots$
step n	$\alpha\beta^n$	$\beta^n$

hits again the detector's walls. With this step definition  $\alpha$  e  $\beta$  are easily calculable, in fact the photon will have at each step:

- a probability  $f$  to be detected  $\Rightarrow \alpha = f$ ;
- a probability  $1 - f$  to hit the non active internal surface and a probability  $R(1 - f)$  to be regenerated  $\Rightarrow \beta = R(1 - f)$ ;

It is now possible to calculate the optical efficiency of the detector using equation 4:

$$\epsilon_{opt} = F(Q = 1, f, R(1 - f)) = \frac{f}{1 - R(1 - f)} \quad (5)$$

In a realistic situation the PSD window has non trivial optical characteristics, that is a transmissivity  $T_w \neq 1$  and a reflectivity  $R_w \neq 0$ . In this case one has:

$$\begin{aligned} \alpha &= T_w f \\ \beta &= R(1 - f) + R_w f \end{aligned} \quad (6)$$

$$Q = 1$$

where the term  $R_w f$  in the definition of  $\beta$  takes into account the regeneration probability on the PSD's window and:

$$\epsilon_{opt} = F(Q = 1, T_w f, R(1 - f) + R_w f) = \frac{T_w f}{1 - R(1 - f) - R_w f} \quad (7)$$

Equation 7 has been derived for a spherical detector, but it can be safely considered a very good approximation for all regular box shapes and, more generally, for all the cases where  $\alpha/T_w$  can be (roughly) identified with  $f$  (PSD surface coverage).

In a even more general experimental situation the scintillation detector could host more than one PSD. In this case one should consider one PSD per time and calculate the optical efficiency (equation 7) with respect to it. For the reflectivity of the remaining part of the cell one should take the average reflectivity of the non active surface and of the remaining PSDs' windows, weighted by their relative surface coverage. The same procedure should be repeated for each one of the PSDs in the cell and the total optical efficiency is obtained by summing up all the individual efficiencies.

## 2.2 Rayleigh scattering and photon absorption.

An ideal scintillation medium is perfectly transparent to its own radiation and thus the emitted photons propagate unabsorbed along straight trajectories between a reflection and the other. In practice this is never the case and photons have a finite probability of being absorbed or elastically scattered (Rayleigh scattering) while traveling across the scintillator because of the presence of some contaminant(s) or because of the intrinsic molecular/atomic structure of the medium. The model developed above allows to include these effects and disentangle them from reflections in an extremely clean way. Consider a cell filled with an uniform scintillator. Assume that  $\alpha_0$  and  $\beta_0$  are the detection and regeneration probabilities calculated with respect to one possible step definition *in absence of scattering/absorption*. If, instead, these effects are present, the step definition needs to be opportunely modified and the detection and regeneration probabilities,  $\alpha$  and  $\beta$ , recalculated. If the photon interacts in the scintillator it can be either scattered or absorbed. Absorption kills the photon while scattering *regenerates* it more or less in the same way as a reflection does. Following this consideration the step definition needs only to be slightly enlarged to include the case that an interaction can stop the current step and start a new one. In order to calculate  $\alpha$  e  $\beta$  the following quantities need to be defined:

- the *effective interaction* length,  $\tilde{\lambda}$ :

$$\frac{1}{\tilde{\lambda}} = \frac{1}{\lambda_R} + \frac{1}{\lambda_A} \quad (8)$$

where  $\lambda_R$  and  $\lambda_A$  are the Rayleigh scattering length and the absorption length respectively;

- the average probability,  $U_{RA}$ , that a photon randomly produced inside the detector reaches the end of the step, as defined in *absence* of scattering/absorption, without interactions.

The detection and regeneration probabilities (per step) of the photon are then:

$$\alpha = U_{RA}\alpha_0$$

$$\beta = U_{RA}\beta_0 + (1 - U_{RA})\frac{\tilde{\lambda}}{\lambda_R} \quad (9)$$

In the definition of  $\beta$  the term  $U_{RA}\beta_0$  accounts for the probability that the photon reaches *untouched* the boundary surface of the cell and is regenerated by reflection while the term  $(1 - U_{RA})\frac{\tilde{\lambda}}{\lambda_R}$  accounts for the probability that the photon interacts before reaching the boundary surface and is regenerated by scattering.

From equation 4 the optical efficiency will be:

$$\epsilon_{opt}^{RA} = \frac{U_{RA}\alpha_0}{1 - [U_{RA}\beta_0 + (1 - U_{RA})\frac{\tilde{\lambda}}{\lambda_R}]} \quad (10)$$

after some algebra one finds that:

$$\epsilon_{opt}^{RA} = \frac{\alpha_0}{Q - \beta_0} \quad (11)$$

where:

$$Q = \frac{1 - (1 - U_{RA})\frac{\tilde{\lambda}}{\lambda_R}}{U_{RA}} \quad (12)$$

Equation 11 generalizes in an elegant way the case of photons traveling in a perfectly transparent medium: all the effects related to scattering/absorption are contained in the term  $Q$  and can be treated/calculated separately. For this reason it seems to be appropriate the notation  $F(Q, \alpha_0, \beta_0)$ , where all the three terms that contribute to the *optical efficiency* are expressly indicated.

The term  $Q$  can be calculated for the interesting case of the detector described in section 2. One can write in an almost general way that:

$$U_{RA} = \int_0^\infty P(x)e^{-\frac{x}{\tilde{\lambda}}}dx \simeq \int_0^{\tilde{L}} P(x)e^{-\frac{x}{\tilde{\lambda}}}dx \quad (13)$$

where  $\tilde{L}$  is some characteristic linear dimension of the detector and  $P(x)$  is the probability density distribution of the distances that a photon would travel in absence of interactions between two reflections (or between one reflection and absorption/detection). To be consistent with the case of regular solids one can define  $\tilde{L} = 6V/S$  with  $V$  the volume and  $S$  the boundary surface of the detector<sup>3</sup> and as a first order approximation one can choose  $P(x)$  to be uniform between 0 and  $\tilde{L}$ , so that:

$$U_{RA} = \int_0^{\tilde{L}} \frac{1}{\tilde{L}} e^{-\frac{x}{\tilde{\lambda}}}dx = \frac{\tilde{\lambda}}{\tilde{L}} (1 - e^{-\frac{\tilde{L}}{\tilde{\lambda}}}) \quad (14)$$

---

<sup>3</sup>This gives  $\tilde{L} = 2R$  for the sphere,  $\tilde{L} = L$  for a cube and  $\tilde{L} = 2R$  for a square cylinder.

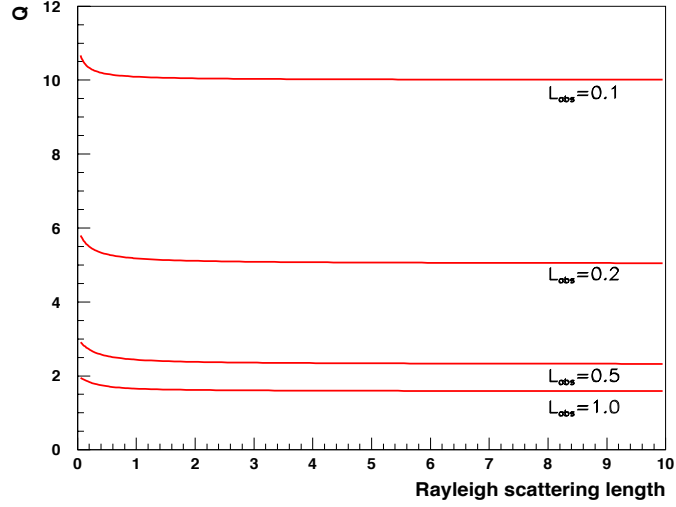


Figure 1: *The term  $Q$  as a function of the Rayleigh scattering length in units of  $\tilde{L}$  for few values of the absorption length  $L_{abs}$  (also in units of  $\tilde{L}$ ).*

and  $Q$  is obtained by substituting this value to  $U_{RA}$  in equation 12. In figure 1 it is shown the plot of the term  $Q$  as a function of the Rayleigh scattering length for fixed values of the absorption length, both in units of  $\tilde{L}$ .  $Q$  is only weakly dependent on the normalized Rayleigh scattering length and visible effects can be seen only when it is smaller than one.<sup>4</sup> On the other side the dependence on the (normalized) absorption length is much stronger and for  $L_{abs} = 1$  the term  $Q$  is already near to 2.

### 3 Monte Carlo tests of the model.

The predictions of this simple toy model have been compared with the results of Monte Carlo simulations of specific scintillation detectors. In order to directly use the formulas found for the example developed in section 2 a scintillator contained in a cubic box is firstly considered. The cubic shape has been chosen because it has a good degree of symmetry, but at the same time is sufficiently far from a sphere, that represents the best approximation of the hypotheses that have been used and thus can be a good benchmark for the toy model.

The cube is assumed to have a side of length  $L = 10 \text{ cm}$  and is observed by one PSD with circular flat window. The PSD window is positioned exactly in the middle of one

<sup>4</sup>It is implicit that the Rayleigh scattering can influence absorption only if the absorption length is different from zero, otherwise it has no effect ( $Q=1$ ).



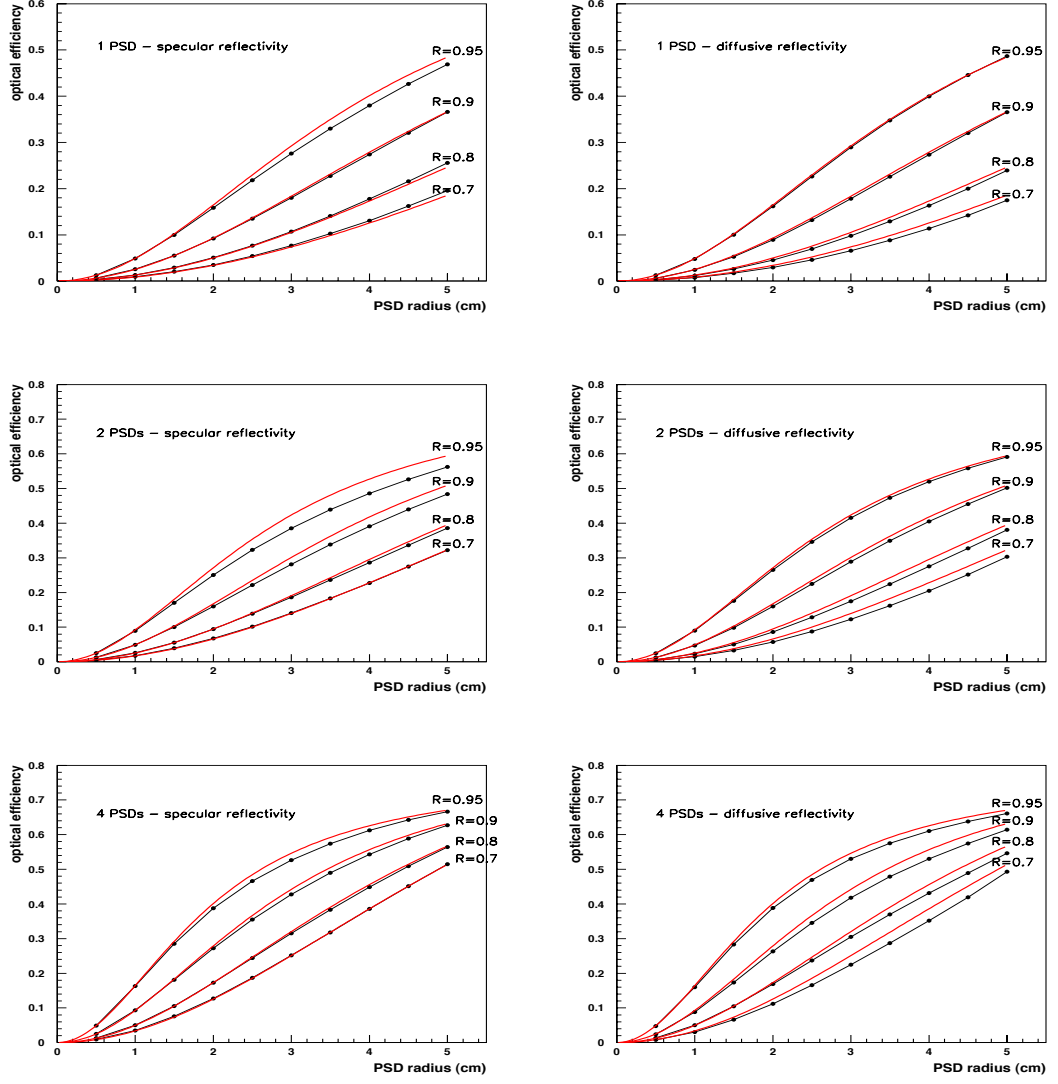


Figure 2: *Top: Optical efficiency for the cubic scintillator with one (Top), two (Middle) and four (Bottom) PSD(s). Specular reflectivity on the left and diffusive reflectivity on the right. Black dots represent the results of Monte Carlo simulations, while red lines are the model predictions.*

of the cube's faces and is assumed to have a reflectivity  $R_w = 0.3$  and a transmissivity  $T_w = 0.5$ , while the reflectivity  $R$  of the internal non active surface is varied between 0.70 and 0.95. The radius of the PSD window is varied between 0.5 cm and 5.0 cm. The optical efficiency of the detector,  $\epsilon_{opt}$ , for any given configuration of the parameters is evaluated by randomly extracting a point inside the cube and generating from it a huge number of photons ( $10^5$ ) with direction uniformly distributed in space. This procedure is repeated for  $10^5$  times and each time the fraction  $N_{det}$  of photons transmitted across the PSD window with respect to generated ones is stored.

The average fraction of *detected* photons is determined by fitting the distribution of  $N_{det}$  with a Gaussian function and taking its central value.

The outcomes of the simulations are displayed as black dots in figure 2, Top. Two cases have been separately considered: the case of completely specular reflections (left) and the case of completely diffusive (lambertian) reflections (right). To estimate the detection efficiency equation 7 has been used (red lines in figure 2), where  $f$  is the fraction of the cube's surface occupied by the PSD window.

The simulation has been repeated for the same cubic cell but with *two* and *four identical PSDs* installed on different non-adjacent faces of the cube. The results, together with the predictions of equation 7 are shown in figure 2 (Middle and Bottom respectively).

In the case of two PSDs, equation 7 has been computed for a single PSD with a reflectivity  $\tilde{R}$  for the remaining internal surface of:

$$\tilde{R} = \frac{(1 - 2f)R + fR_w}{1 - f} \quad (15)$$

The optical efficiency of the system is two times the one of the single PSD, since the two PSDs are identical. In the case of four PSDs an analogous calculation has been performed. For all the examined cases small differences between specular and diffusive reflectivity are found. This simple model very well reproduces the results of the Monte Carlo simulations and discrepancies at the level of few percent are found.

The dependence of the optical efficiency on the window's reflectivity  $R_w$  has been tested with a dedicated simulation of the cubic cell with one PSD. The PSD radius is fixed at 4 cm, the (specular) reflectivity of the walls at 0.90 and  $R_w$  is varied between 0.1 and 0.9. The transmissivity of the window  $T_w$  is set at  $1 - R_w$ . The results are shown in figure 3. Also in this case equation 7 (red line) cleanly reproduces the outcomes of the simulation (black dots).

The predictions of equation 11, that is in the case Rayleigh scattering and absorption effects are present, have been tested again with the cubic scintillator with one PSD installed. In this case the (specular) reflectivity of the non active surface is set at 0.95 and the circular window radius of the PSD varied between 0.5 cm and 5 cm. As a first the Rayleigh scattering length (that typically has very small effects, see section 2.2) is fixed at 10 cm and the absorption length is varied between 10 cm and 400 cm. Results are shown in figure 4 left. With the same choice of parameters, except the absorption length and PSD radius fixed respectively at 50 cm and 4 cm, the dependence of the optical

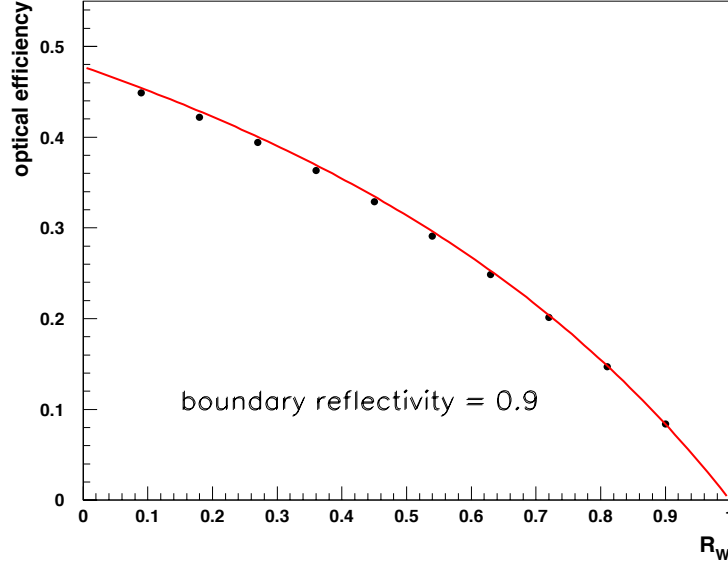


Figure 3: *Dependence of the optical efficiency on the wall's reflectivity  $R_w$ . The cubic cell has one PSD and the reflectivity of the walls has been fixed at 90%. The transmissivity of the window is  $T_w = 1 - R_w$ . Black dots comes from simulation and red line represents the prediction of the model.*

efficiency from the Rayleigh scattering length has been separately tested by varying it from 2 cm to 40 cm. Results are shown in figure 4 right. It is impressive how the term  $Q$  calculated from equations 12 and 14, that are derived from very general considerations, accounts for the effects of scattering and absorption over all the scanned lengths and with this level of accuracy.

The tests have been repeated with many different choices of the optical parameters (boundary surface reflectivity, PSD window dimensions, Rayleigh scattering length, absorption length, ...) and the agreement between MC data and model predictions has always been found at the level shown here.

For the last and most challenging test a scintillation detector with a poor degree of symmetry is considered. The active medium is contained in a parallelepiped box with internal dimensions: 10 cm x 10 cm x 30 cm ( $l \times w \times h$ ). Two identical PSDs with circular window are installed on the two opposite square (10 cm x 10 cm) faces of the box. The reflectivity of the passive internal surface is varied between 0.8 and 0.97 (specular or diffusive) and the radius of the PSD windows between 0.5 and 5 cm. The windows' reflectivity is set at 0.3 and its transmissivity at 0.5 (as for the cubic scintillator). Again the optical efficiency is evaluated with dedicated MC simulations and compared with the prediction of equation 7 ( $f$  is the PSD coverage of the internal surface). The two cases of specular and diffusive reflectivity are treated separately. The results of these tests are

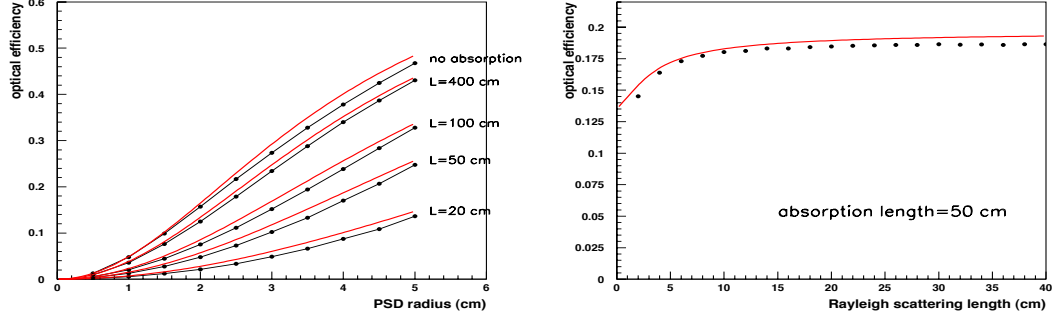


Figure 4: *Left: optical efficiency of the cubic scintillator with one PSD. Rayleigh scattering length = 10 cm. Attenuation length varied between 10 and 400 cm. Right: optical efficiency as a function of the Rayleigh scattering length. The attenuation length is fixed at 50 cm, the PSD window has a radius of 4 cm and the reflectivity of the non instrumented surface is 0.95. Black dots represent the results of the simulation while the red line is the prediction of the model.*

shown in figure 5. Surprisingly also in this case the agreement between MC outcomes and model predictions is satisfactory good. In fact, for specular reflectivity above 0.9 discrepancies at the level of few percent are found, while in all other cases they are of the order of 10%.

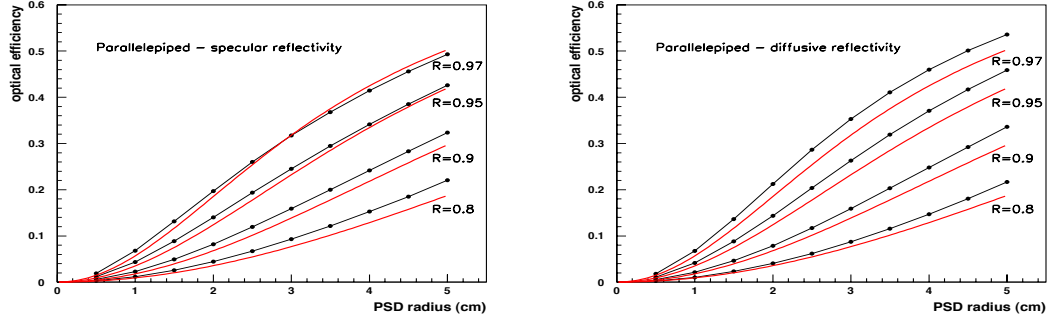


Figure 5: *Optical efficiency of the parallelepiped scintillation detector. Left: specular reflectivity. Right: diffusive lambertian reflectivity. Black dots represent the results of Monte Carlo simulations, while red lines are the model predictions.*

## 4 Adding a wavelength shifter to the scintillator.

Some materials emit scintillation photons in the ultraviolet region of the electromagnetic spectrum. In these cases it is a common practice to dilute wavelength shifting substances (WLS) in the scintillator ([4], [5]) that absorb ultraviolet light and re-emit visible photons easily detectable with glass windowed photomultipliers. The formulas obtained in the previous sections for the evaluation of  $\epsilon_{opt}$  are still perfectly applicable with the small modification of introducing an overall efficiency for the conversion of photons (typically near to one). If it is not possible to dilute any WLS in the scintillator an alternative solution consists in depositing it (by vacuum evaporation, spraying, painting, ...) on the internal surface of the detector, PSDs' windows included. This is the case, for instance, of double phase argon detectors ([6], [7]), where the stringent requests on the liquid purity limits the admissible amount of diluted contaminants to tens of ppb of electronegative substances and hundreds of ppb of non-electronegative ones ([8], [9]). This situation can be well handled with the ideas developed above, but some particular care must be taken.

Consider a scintillation detector with only one PSD (situation easily generalizable to the case of  $n$  PSDs, as shown above). Assume that the scintillator medium is contained in a box of regular shape, so that equation 7, generalized to the case of  $Q \neq 1$ , can be used. Define  $\epsilon_{WS}$  as the shifting efficiency of the non instrumented internal surface and assume that  $R$  is their reflectivity to shifted photons. Define also  $\epsilon_{ws}$  as the shifting efficiency of the PSD window that has a transmissivity  $T_w$ , a reflectivity  $R_w$  (to shifted photons) and that covers a fraction  $f$  of the internal surface of the cell.

The crucial point is here the calculation of the probability that a VUV (Vacuum Ultra Violet) photon reaches the boundary surface of the detector where it is wavelength-shifted. A simple argument (see Appendix A) shows that this probability is equal to the inverse of the term  $Q$  (section 2.2) calculated using the Rayleigh scattering length and the absorption length of VUV photons in the scintillator medium ( $Q_{VUV}$ ). To evaluate  $\epsilon_{opt}$  it is necessary to consider that:

- the probability that a VUV photon reaches the boundary surface of the cell is  $\frac{1}{Q_{VUV}}$ ;
- the probability that the photon is down-converted on the window of the PSD is  $\frac{f\epsilon_{ws}}{Q_{VUV}}$ . Since the emission process is isotropic, the probability that it is *directly* transmitted across the PSD window is roughly half of the total:

$$\epsilon_{opt}^{direct} = \frac{f\epsilon_{ws}}{2Q_{VUV}} \quad (16)$$

eventually reduced by a factor that takes into account the absorption of the window. The complementary (half) part is the probability that the photon is sent back in the cell;

- the probability that the photon is down-converted on the non instrumented surface of the cell is  $\frac{(1-f)\epsilon_{WS}}{Q_{VUV}}$ . The shifted photon will propagate inside the cell and will be detected with a probability  $F(Q_{vis}, T_w f, R(1-f) + R_w f)$ . Where  $Q_{vis}$  is calculated using Rayleigh scattering and absorption lengths for *visible* photons;
- the probability that the photon is *indirectly* detected is then:

$$\epsilon_{opt}^{indirect} = \left( \frac{f\epsilon_{ws}}{2Q_{VUV}} + \frac{(1-f)\epsilon_{WS}}{Q_{VUV}} \right) F(Q_{vis}, T_w f, R(1-f) + R_w f) \quad (17)$$

Here we take into account that the photon can come from the PSD's window or from the inactive surface.

In conclusion the total detection probability is:

$$\epsilon_{opt} = \epsilon_{opt}^{direct} + \epsilon_{opt}^{indirect} = \frac{T_w f \epsilon_{ws}}{2Q_{VUV}} + \left( \frac{f\epsilon_{ws}}{2Q_{VUV}} + \frac{(1-f)\epsilon_{WS}}{Q_{VUV}} \right) F(Q_{vis}, T_w f, R(1-f) + R_w f) \quad (18)$$

It is interesting to notice here that because of the term  $\frac{(1-f)\epsilon_{WS}}{Q_{VUV}}$  this is not a priori a monotonically increasing function of the PSD coverage  $f$ . It is then possible that for a certain set of optical parameters the maximum value of the optical efficiency is not reached with a total PSD coverage of the internal surface, i.e.  $f = 1$ , but for some optimal value that can be found by maximizing  $\epsilon_{opt}$  (equation 18) with respect to  $f$ .

Table 2: *Parameters used to evaluate the LY of the detector described in [10].*

photon yield	$N_\gamma=40$ photons/keV [11]
photocathodic coverage	$f=13\%$
transmissivity of PMT window	$T_w=0.94$ [12]
reflectivity of PMT window	$R_w=0$
conversion efficiency of PMT	$\epsilon_{PSD}=28\%$
no absorption of VUV photons	$Q_{VUV}=1$
no absorption of visible photons	$Q_{vis}=1$
conversion efficiency of passive surface	$\epsilon_{WLS}=1$ [13]
conversion efficiency of PMT window (no shifter)	$\epsilon_{wls}=0.$
reflectivity of passive surface (reflector+TPB)	$R=0.95$ [14]

Equation 18 has been used to evaluate the LY of the scintillation detector described in full detail in [10]. The scintillating medium is liquid Argon that is contained in a cylindrical PTFE cell ( $h=9.0$  cm and  $\phi=8.4$  cm) and is observed by a single 3" photo-multiplier. Liquid Argon is an abundant scintillator ( $\sim 40$  photons/keV) but photons

are emitted in the VUV region of the electromagnetic spectrum ( $\lambda=128$  nm) and need to be wavelength shifted to be detected with the installed photomultiplier (synthetic silica window - cutoff around 200 nm). For this reason the internal surface of the cell is completely covered with a reflective foil deposited with Tetra Phenyl Butadiene (TPB), that is an extremely efficient shifter with emission spectrum peaked around 420 nm [13] [14]. The parameters used to evaluate the LY of the detector are summarized in table 2.

For  $\epsilon_{PSD}$  the product of the photocathode quantum efficiency averaged over the TPB emission spectrum (29.5%) and of the photoelectron's collection efficiency at first dynode (95%) has been taken. Given the refractive index of LAr ( $n_{LAr}(420\text{ nm})=1.25$  [15]) and of the synthetic silica window ( $n_{window}(420\text{ nm})=1.46$  [16]) reflection of photons at the interface can be neglected in good approximation, and hence  $R_w$  has been set to zero. The LY resulting from equations 3 and 18 is 6.9 phel/keV, in perfect agreement with the measured value of  $7.0\text{ phel/keV} \pm 5\%$ .

## 5 Conclusions.

A toy-model for the estimation of the light yield of a scintillation detector based on very simple hypotheses has been developed. It has been shown how to include the effects related to Rayleigh scattering and absorption of the photons.

The model has been benchmarked with the outcomes of the Monte Carlo simulation of a cubic scintillator observed by one, two or four PSDs and has demonstrated an accuracy at the level of few percent. An additional Monte Carlo test with a parallelepiped detector with a poor degree of symmetry has been performed. Even in this case the model predictability has resulted surprisingly good and an agreement with Monte Carlo outcomes better than 10% has been obtained.

The model has been also applied to the estimation of the light yield of a real liquid Argon scintillation detector and a value of 6.9 phel/keV has been found, perfectly compatible with the measured value of  $7.0\text{ phel/keV} \pm 5\%$ .

The formulas here reported can be adequate in all those cases a quite robust estimation of the light yield of simple scintillation detectors is needed. It can result very useful in the optimization of the design of the detector since the dependence of the light yield from the optical parameters is completely explicit. Even in presence of a Monte Carlo simulation of the detector the model can be useful to cross-check and validate its predictions.

Furthermore the formulas found for the examples treated along the paper can be directly used in many real applications.

## References

- [1] W.R. Leo, *Techniques for Nuclear and Particle Physics Experiment*, Springer (1994).
- [2] G.F. Knoll, *Radiation Detection and Measurement*, J. Wiley (2000).
- [3] J.B. Birks, *The Theory and Practice of Scintillation Counting*, Pergamon Press (1964)

- [4] Borexino Coll., *The Borexino detector at the Laboratori Nazionali del Gran Sasso*, Nucl. Instr. and Meth. A Volume 600, Issue 3, 11 March 2009, Pages 568-593
- [5] LVD Coll., *The large-volume detector (LVD) of the Gran Sasso Laboratory*, Nuovo Cimento, C : 9 (1986) , pp.237-261
- [6] WArP Coll., *The WArP experiment*, Journal of Physics: Conference Series **203** (2010) 012006.
- [7] ArDM Coll., *The ArDM experiment*, <http://neutrino.ethz.ch/ArDM>
- [8] WArP Coll., *Effects of Nitrogen contamination in liquid Argon*, JINST **5** (2010) 06003.
- [9] WArP Coll., *Oxygen contamination in liquid Argon: combined effects on ionization electron charge and scintillation light*, JINST **5** (2010) 05003.
- [10] WArP Coll., *Demonstration and Comparison of Operation of Photomultiplier Tubes at Liquid Argon Temperature*, arXiv:1108.5594 (2011).
- [11] T. Doke, *Fundamental properties of liquid Argon, Krypton and Xenon as Radiation detector media*, Portugal Phys. **12** (1981), 9.
- [12] Hamamatsu Photonics K. K., *PHOTOMULTIPLIER TUBES. Basics and Applications.*, Edition 3a (2007).
- [13] W.M.Burton and B.A.Powell, *Fluorescence of Tetraphenyl-Butadiene in the Vacuum Ultraviolet*, Applied Optics, 12 (1973), 87.  
D.N. McKinsey et al., *Fluorescence efficiencies of thin scintillating films in the extreme UV spectral region* Nucl. Instr. and Meth. B, **132** (1997), 351.
- [14] E. Nichelatti et al., *Optical characterization of organic light-emitting thin films in the UltraViolet and Visible spectral ranges*, ENEA Tech. Report RT/2010/31/ENEA (2010).
- [15] ICARUS Coll. *Detection of Cherenkov light emission in liquid argon* Nucl. Instr. and Meth. A 516 (2004) 348363.
- [16] <http://www.sciner.com/Opticsland/FS.htm>

## A VUV photons absorption.

Consider a scintillation detector and assume that photons suffer Rayleigh scattering and absorption. The optical efficiency of the detector is (equation 11):

$$\epsilon_{opt} = \frac{\alpha_0}{Q - \beta_0} \quad (19)$$



where  $\alpha_0$  and  $\beta_0$  are the detection and regeneration probabilities in absence of scattering/absorption. If, unlike what has been done in section 2.2, the step definition is not changed and remains as the photon's propagation between two subsequent reflections, the detection probability will be  $q\alpha_0$  and the regeneration probability  $q\beta_0$ , where  $q$  is the photon's surviving probability along the step. Consequently the optical efficiency can be written as:

$$\epsilon_{opt} = \frac{q\alpha_0}{1 - q\beta_0} = \frac{\alpha_0}{1/q - \beta_0} \quad (20)$$

Comparing equations 11 and 20:

$$q = \frac{1}{Q} \quad (21)$$